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# On local and global measurements of the speed of light on rotating platforms.

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## Abstract

The paper shows that, conceptually and operationally, the speed of light as measured locally in the inertial comoving frame of a point on the rim of a rotating disk, is different from the one measured globally for a round trip along the rim, obtained dividing the length of the rim (as measured in the "relative space" of the disk) by the time of flight of the light beam (as measured by a clock at rest on the disk). As a consequence, contrary to some recent claims, the anisotropy found in the global value, obtained by the above procedure, in no way conflicts with the local isotropy, and the internal consistency of the special relativity theory remains unchallenged.

## I. Introduction

In 1997 Franco Selleri,<sup>1</sup> in his long quest for inconsistencies in the special relativity theory (SRT), pointed out a paradox concerning the speed of light as measured on board a rotating disk. Actually his point treats the speed of light along a closed circuit encircling the rotation axis: when the platform is moving, the speed obtained dividing the length of the contour by the time of flight, as measured in the "relative space" of the disk (defined in Sec. 3) by an observer at rest on the platform, is different whether measured in the rotation sense or in the opposite sense. More explicitly, suppose a light beam is sent along the rim in the rotation sense and another one in the opposite sense; then measure the average velocities of both beams for a complete round trip, i.e. the ratio between the length of the path and the times of flight read on a clock at rest on the rim, and call them  $c_+$  and  $c_-$ ; then the ratio  $\rho = c_+/c_-$  differs from 1.

Since a rotating reference frame is not inertial, this anisotropy of the light propagation is not, on itself, a surprising result; however Selleri notices that when letting the platform's radius  $R$  go to infinity and the angular speed  $\omega$  go to 0 in such a way that the peripheral speed  $\omega R$  of the turntable remains constant, the ratio  $\rho$  too keeps a constant value. However in the limit of infinite radius the uniform rotation becomes a uniform translation, i.e. the local reference frame becomes inertial. Here, according to SRT, the speed of light is assumed to be exactly the same in any direction (since all inertial frames are assumed to be optically isotropic); hence  $\rho$  must be strictly 1. This alleged discontinuity in the behavior of  $\rho$ , under such limit process, is the core of what we could call Selleri's paradox.

This issue has already been discussed elsewhere,<sup>2</sup> showing that a full 4-dimensional relativistic treatment of the problem of the rotating platforms avoids any discontinuity or inconsistency whatsoever, since the speed of light, consistently defined, turns out to be exactly the same both clockwise and counterclockwise, just as in an inertial reference frame.

However, though the 4-dimensional geometric point of view is clear and consistent, nothing prevents from considering the problem from a different viewpoint, rather natural for an observer living on the rotating disk. Then some doubt is apparently allowed,<sup>1,3</sup> since the ratio  $\rho$ , when measured by means of meter rods and clocks (or rather a single clock) at rest on the

platform, actually differs from 1.

The root of Selleri's paradox can be identified in the basic assumption - founded on the homogeneity of the disk along the rim - that the "global ratio"  $\rho = c_+/c_-$  of the average light velocities for complete round trips, coincides with the "local ratio"  $\rho_o$  of the forward and backward light velocities. We shall however show, analyzing the actual measurement procedures of the velocities in both cases, that  $\rho$  cannot in general be assumed to equal  $\rho_o$ , contrary to the claim by Selleri. In fact,  $\rho$  does not depend on the criterium for simultaneity adopted along the rim, as Selleri correctly points out;  $\rho_o$  does instead strictly depend on the local simultaneity criterion. The two ratios, which Selleri labels by the same letter  $\rho$ , refer to two different kinds of measurements, so that there is no point in comparing them: they are and remain different, whatever the size of  $R$  is, be it finite or infinite, with no harm for SRT. That this was the weak point of Selleri's argument has been already remarked also by Budden.<sup>4</sup>

In sect. 2 the four-dimensional approach considered in ref. [2] is briefly reexamined. Sect. 3 discusses two possible alternative definitions of space of the platform along the rim, and compares the Minkowskian and the operational approach to the interpretation of the measurements of space and time intervals on board the rotating disk. Sect. 4 draws the general conclusions.

## II. Constancy of the speed of light in Minkowski space-time

On a formal point of view, the SRT is the description of a four-dimensional manifold, whose geometrical structure is uniquely determined by two principles: the Einstein relativity principle and the principle of constancy of the (one way) velocity of light in vacuum<sup>1</sup>. As well known, such manifold is the familiar Minkowski spacetime, in which the time evolution of any massive particle is described in terms of a world line  $\gamma_m$  which lies everywhere inside the light cone associated with any point of  $\gamma_m$ .

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<sup>1</sup>Of course, the axiomatic basis of the SRT is not completely established by the two principles mentioned above, but also embodies the so-called "principle of locality", which states the local equivalence of any accelerated reference frame with a momentarily comoving inertial frame.

This can be visualized in a standard spacetime diagram (in which space and time are measured by the same unities and the coordinate lines are drawn orthogonal to each other) as a world line whose slope, although variable, is everywhere greater than  $45^\circ$ . Only massless particles, particularly photons, are described by null world lines, i.e. by world lines whose slope, in the graphic representation, is always  $45^\circ$ : any light beam in free spacetime is described by a  $45^\circ$  slanting straight line, which can be regarded as a generator of the light cone. This is a geometrical expression of the principle of constancy of the one way velocity of light in minkowskian spacetime. The interaction of the light beam with a mirror may change the space direction of propagation of the beam, curving the trajectory in space and the world line in spacetime, *without affecting its slope*. As a consequence, when a light beam is lead to move along the rim of a rotating disk, grazing a cylindrical mirror, its world line in  $2+1$  dimensions turns out to be a "null helix" wrapped around the world tube of the disk and keeping everywhere a  $45^\circ$  slope.

A  $2+1$  geometrical analysis of the Sagnac effect (see<sup>2</sup>) shows how and why the times of flight for co-rotating and counter-rotating beams are different, *although their world lines are helices of constant ( $45^\circ$ ) slope*; or, phrasing it differently, *although their velocities are the same - namely  $c$  - in any inertial frame, in particular in the local inertial comoving frame at any point of the rim*.

To sum up, the special relativistic assumption of the constancy of the slope of the world lines of light does not lead to inconsistencies or unphysical discontinuities. On the operational point of view, this means that, in the framework of SRT, the apparent global anisotropy of the propagation of light along the rim is perfectly compatible with the local isotropy, contrary to Selleri's assumption.

### **III. Actual measurements of the speed of light**

Once the internal consistency of the geometry of Minkowskian spacetime has been established again, it still remains to confront it with the operational procedures an observer at rest on the rotating disk uses, in order to attribute actual values to the physical quantities of interest. The problem is that any measurement concerning the geometry of the disk and the synchronization of clocks on it is a well defined set of physical and mathematical operations on

an extended region of space (in particular along the rim), whereas in a rotating frame special relativistic formulae are merely local: any result obtained by extrapolating them globally cannot be considered as a pure consequence of SRT, but depends on some (usually hidden) further assumptions. In our opinion, the presence of recurrent contradictions and paradoxes simply underlines the arbitrariness of such extrapolations, from local to global.

Now, the obvious operational way to define and determine the (one way) speed of a (massive or not) moving object is to measure the length of a given travel and the time it takes, then divide the former by the latter. Of course this procedure determines the slope of the world line of the moving object only when the measurements are local (infinitesimal extension of the space and time intervals); finite measurements can yield the slope only in very special cases (constant slope world lines).

In the case of uniform rotation and of light travelling along the rim of the rotating disk, the slope of the light world line is constant as well as that of the observer's one; as a consequence, it can be determined not only by (a sequence of) local measurements of space and time intervals, performed in the local comoving frames, but also by global measurements of space and time intervals referred to a complete (either co-rotating or counter-rotating) round trip. However, in the second case the operational procedure should be carefully defined, because of the presence of some unavoidable conventional extrapolations, as pointed out before. More precisely: (i) the measure of the length of a complete round trip depends on the definition of "space on the platform", at least along the rim; (ii) the time taken by the light beam for a complete round trip is an observable quantity (it is the proper time lapse of a single clock), but the impossibility of a global synchronization along the rim could require a suitable correction. The form of the correction is imposed by the space-time geometry, according to the particular definition chosen for the space of the platform (see later).

Among the many possible definitions of "space on the platform along the rim", we consider in particular the following two: (i) the "space of locally Einstein simultaneous events", defined as the set of events along the rim such that any nearby pair of them are simultaneous according to the Einstein criterium; (ii) the "relative space"  $S := T/I$ , defined as the quotient of the world tube  $T$  of the disk by the congruence  $I$  of the word lines of the points of the disk.

The former is obtained extrapolating the local Einstein synchronization procedure to the whole rim of the disk. This space coincides with the space-like helix  $\gamma_S$  considered at the beginning of Sec. 4 of ref. 2, and is everywhere Minkowski-orthogonal to the time-like helices corresponding to the world lines of the points of the rim.

The latter turns out to be the space of locations on the disk, regardless of any kind of synchronization: "two points of spacetime which lie on the same disk word line ... are identified in the relative space".<sup>5</sup> This space seems rather artificial on a Minkowskian point of view, but it should appear quite natural for the observer on the platform, since the space spanned by meter sticks arranged on the platform by this observer is precisely the "relative space" of the disk.

Notice that we used both spaces, namely: the "space of Einstein locally simultaneous events" when we adopt a Minkowskian approach, like in the main part of<sup>2</sup>; and the "relative space" when we adopt an operational approach, like in sect. 5 of<sup>2</sup> and everywhere in.<sup>6</sup>

### A. *Minkowskian approach*

In this approach, the "space of locally Einstein simultaneous events" along the rim coincides with the space-like helix  $\gamma_S$  considered before; the important point is that  $\gamma_S$  is not a circumference (this is true only in the absence of rotation), but an open line whose slope depends on the rotation velocity. Now, the proper length of an open line is not a uniquely defined entity; we showed in particular in<sup>2</sup> that the geometry of Minkowskian spacetime imposes different lengths for the portion of  $\gamma_S$  covered by the co-rotating and by the counter-rotating light beams in a complete round trip. The difference in these two lengths turns out to be

$$\delta s_{\gamma_S} = \frac{4\pi(\omega R)}{c\sqrt{1 - (\omega R)^2/c^2}}R \quad (1)$$

which exactly coincides, dividing by  $c$ , with the difference in time of flight along the two round trips (see eq. (2) later, which is consistent with the Sagnac effect). As a consequence, this definition of space ensures the equality of the global speed of light both for the co-rotating and the counter-rotating light beams, restoring the isotropy of light propagation.

We point out that this definition of space is the only one which can insure the equality between global and local velocities, i.e. between the "global ratio"  $\rho$  and the "local ratio"  $\rho_o$ : this agrees with Selleri's assumption, but both ratios equal exactly 1, with no harm for the SRT.

## B. *Operational approach*

In this approach, the "relative space"  $S$  along the rim allows the observer at rest on the platform to consider a unique length for the rim of the disk (see sect. 5 of<sup>2</sup> and<sup>6</sup>). The measure of the two round trip times is obtained by one single clock (no need for special synchronization procedures), and gives two different results. In particular, the difference in time between the two round trips is

$$\delta\tau = \frac{4\pi(\omega R)}{c^2\sqrt{1-(\omega R)^2/c^2}}R \quad (2)$$

which is an expression of the Sagnac effect,<sup>78</sup>. In this case, the observer can draw the following conclusions, on the basis of his measurements of space and time on the platform and without any knowledge of Minkowskian spacetime structure (see,<sup>2</sup> sect.5): (i) the platform on which he lives is rotating, and the desynchronization  $\delta\tau$  of a pair of clocks, after slow round trips in opposite directions, is a measure of the speed of this rotation; (ii) the durations of travels along the closed path are not uniquely defined and, to obtain reliable measures of them, the readings of clocks must be corrected by a quantity  $\pm\delta\tau/2$  to account for the desynchronization effect, which is the same result obtained by Bergia and Guidone<sup>9</sup>; (iii) as a consequence of this correction, the speed of light is actually the same both forward and backward.

On the other hand, if the readings are used without any theoretical correction, the global measurement actually gives an anisotropic result at all radii (as far as  $\omega R \neq 0$ ); but this procedure, which is the one proposed by Selleri, cannot prove his basic assumption, only founded on the homogeneity of the disk along the rim, that the "global ratio"  $\rho$  coincides with the "local" one  $\rho_o$ .

In fact, the measurements performed by the observer on the platform in order to calculate the two ratios are completely different. The value of the

"global ratio" turns out to be

$$\rho = \frac{c_+}{c_-} = \frac{c - \omega R}{c + \omega R} \quad (3)$$

and depends on the measurement of a difference of proper times read on a single clock. This measurement is independent from any assumption about synchronization.

On the contrary, the "local ratio"  $\rho_o$  depends: (i) on the measurement of two infinitesimal lengths (forward and backward) in the local comoving frame; (ii) on the readings of three clocks (placed at the starting point of the light beams and at the arrival points, in opposite directions), Einstein synchronized in the local comoving frame. If Einstein synchronization is used, the "local ratio"  $\rho_o$  is exactly 1, and cannot be identified with the "global ratio"  $\rho$ , which differs from 1.

One could object that also a local measurement of the light velocity can be performed by means of a single clock, when the light beam is reflected by a mirror placed at an infinitesimal distance from the source (two ways average light speed). But in this case - that is what is usually made in actual experiments, like e.g. Michelson-like experiments - the difference of measurement procedures is still more evident: the global method, which measures two one-way velocities of two light beams performing two complete round trips along a closed path in opposite directions, cannot be used in the local inertial frame, in which only the two ways light speed is measurable. So the "global ratio"  $\rho$  only is an observable; the "local ratio"  $\rho_o$  is not. Selleri's assumption is

$$\rho = \rho_o = \frac{c - \omega R}{c + \omega R} \neq 1 \quad \forall \omega \neq 0 \quad (4)$$

This assumption is equivalent to assuming a suitable non Einstein synchronization in the local comoving frame, which could be called "Selleri synchronization", consistent with the condition,<sup>34</sup>:

$$c_+ = c \left(1 + \frac{\omega R}{c}\right)^{-1}; \quad c_- = c \left(1 - \frac{\omega R}{c}\right)^{-1} \quad (5)$$

Such a synchronization requires of course a suitable non Lorentz coordinate transformation, which in turn implies the existence of a privileged frame and the absolute character of synchronization, see,<sup>1,3,10</sup>.

An obvious consequence of eqs. (5) is that light propagates anisotropically in any local comoving frame along the rim, but the observable two ways light speed is again  $c$ . As a consequence, the "Selleri synchronization" does not conflict with known experiments, but conflicts with the standard Einstein synchronization (which assumes that light propagates isotropically in any inertial frame:  $c_+ = c_- = c$ ).

If Selleri's synchronization is used, the SRT is violated; in this case Selleri's paradox only shows that, starting from an assumption violating the SRT, a result violating the SRT follows.

We cannot treat, in the limits of this letter, the question of which synchronization (Einstein or Selleri) is more adequate to the whole experimental and theoretical context: we limit ourselves to claiming that both are consistent and compatible with experiments, but the "serious logical problem in the SRT" declared by Selleri does not exist.

#### **IV. Conclusion**

To sum up our line of thought, we have seen that the direct measurement of the speed of light along a closed path, free of any theoretical corrections, does indeed reveal an anisotropy when the observer is rotating along the contour. It would continue to be so also for a contour of infinitely great curvature radius, though it is impossible to actually perform the experiment.

On the other side local measurements of the speed of light cannot evidence any anisotropy. The global and local ratios between forward and backward light velocities, which Selleri labels by the same letter  $\rho$ , refer to two different kinds of measurements, which cannot be reduced one to the other: they are and remain different, whatever the size of the platform radius is, be it finite or infinite. The two classes of measurements do not overlap and do not reveal, in the framework of the SRT, any internal contradiction.

## References

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<sup>3</sup>F. Selleri, *Teorie alternative alla relatività e natura del tempo*, in press in the book "La scienza e i vortici del dubbio", a cura di L.Conti e M.Mamone, Capria, Ed. Scientifiche Italiane, Napoli (1998)

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<sup>5</sup>T. Budden, private communication; see also R. Geroch, *J. Math. Phys.*, **12**, 918 (1971)

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<sup>9</sup>S. Bergia, M. Guidone, *Found. Phys. Let.* (in press)

<sup>10</sup>F. Selleri, *Found. Phys.*, **26**, 641 (1996)